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DOMINATION IN FUZZY PLANAR GRAPH

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Abstract

Fuzzy Planar Graph is an important subclass of fuzzy graph. In this paper we introduced the concept "Domination in Strong and Isomorphic Fuzzy Planar Graph". It is combination of domination and fuzzy planar graph. We discussed in this paper various properties like domination in fuzzy planar graph (γ_{FP}), domination in strong fuzzy planar graph (γ_{SFP}), domination in isomorphic fuzzy planar graph (γ_{IFP}), domination in fuzzy planar graph with planarity.

Keywords:

Fuzzy graph,
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I. INTRODUCTION

Graph Fuzzy graph and Planar graph are the sub-class of graph theory. Combination of these two graph is called "Fuzzy Planar Graph". One of the fastest growing areas within graph theory is the study of domination.

We discussed the concept of dominating graph were introduced by V.R. Kulli and Bidarhalli Janakiraman[3]. We used the Concept Of Fuzzy Planar Graph with using planarity value is introduced by Sovan Samantha, Anita Pal, Madhumangal Pal [6] and Domination in

Fuzzy Graph is introduced by A.Somasundaram and S.Somasundaram [4]. In this paper we introduced the concept of Domination In Strong, Isomorphic Fuzzy Planar Graph.

II. PRELIMINARIES

Definition: 2.1

A ***finite graph*** is a graph $G = (V, E)$ such that V and E are called vertices and edges finite sets.

Definition: 2.2

An ***infinite graph*** is one with an infinite set or edges or both. Most commonly in graph theory, it is implied that the graphs discussed are finite.

Definition: 2.3 If more than one edge joining two vertices is allowed, the resulting object is a ***multigraph***[1]. Edges joining the same vertices are called ***multiple lines***.

Definition: 2.4

A drawing of a geometric representation of a graph on any surface such that no edges intersect is called ***embedding***.

Definition: 2.5

A graph G is ***planar*** [1] if it can be drawn in the plane with its edges only intersecting at vertices of G. So the graph is ***non-planar*** if it cannot be drawn without crossings.

Definition: 2.6

A $\boldsymbol{fuzzy\ set\ A}$ [2] on a universal set X is characterized by a mapping $m: X \to [0,1]$, which is called the membership function. A fuzzy set is denoted by $A = (X, m)$.

Definition: 2.7

A $\boldsymbol{fuzzy\ graph}$ [2] $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of function $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x)$^$\sigma(y)$ Where $\sigma(x)$ and $\mu(x, y)$ represent the membership values of the vertex x and of the edge (x, y) in G respectively.

Definition: 2.8

The fuzzy graph $G - (V, \sigma, \mu)$, an edge (x, y) is called $\boldsymbol{strong}$ [5] if $\frac{1}{2}\{\sigma(x)$^$\sigma(y)\} \leq \mu(x, y)$ and weak otherwise.

Definition: 2.9

The ***order p*** [4] and ***size q*** [4] of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{x,y \in E} \mu(xy)$.

Definition: 2.10

$N(x) = \{y \in V \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}$ is called the ***neighborhood of x*** and $N[x] = N(x) \cup \{x\}$ is called the ***closed neighborhood of x***[4].

Definition: 2.11

If an edge (x, y) of a fuzzy graph satisfies the condition $\mu(x, y) = \sigma(x) \wedge \sigma(y)$, then this edge is called ***effective edge*** [6].

Definition: 2.12

Two vertices are said to be ***effective adjacent*** [6] if they are the end vertices of the same effective edge.

Definition: 2.13

The ***minimum neighborhood degree*** is denoted by δ_N and ***maximum neighborhood degree*** is denoted by Δ_N

Definition: 2.14

A ***homomorphism*** [6] between fuzzy graph G and G' is a map $h: S \to S'$ which satisfies $\sigma(x) \leq \sigma'(h(x))$ for all $x \in S$ and $\mu(x, y) \leq \mu'(h(x), h(y))$ for all $x, y \in S$ where S is set of vertices of G and S' is that of G'.

Definition: 2.15

An ***isomorphism*** [6] between fuzzy graph is a bijective homomorphism $h: S \to S'$ which satisfies $\sigma(x) = \sigma'\big(h(x)\big)$ for all $x \in S$ and $\mu(x, y) = \mu'(h(x), h(y))$ for all $x, y \in S$.

Definition: 2.16

Let *G be a* ***fuzzy planar graph*** with planarity value f**,** [6] where

$f = \frac{1}{1+\{I_{P_1}+I_{P_2}+\cdots+I_{P_n}\}}$. The range of f is $0 < f \leq 1$.

Here, $P_1, P_2, \ldots . P_n$ be the points intersections between the edges.

In a graph $G = (V, \sigma, E)$, E contains two edges $\mu(a, b)$ and $\mu(c, d)$, which are intersected at a point P.

Strength of the fuzzy edge $I_{(a,b)} = \frac{\mu(a,b)}{\{\sigma(a)^\wedge\sigma(b)\}}$.

The intersecting point at P is $I_P = \frac{I_{(a,b)}+I_{(c,d)}}{2}$.

Results: 2.17

- If there is **no point of intersection** for a geometrical representation of a fuzzy planar graph, then **its fuzzy planarity value is 1**.
- If $\mu(w, x) = 1$ (or near to 1) and $\mu(y, z) = 0$ (near to 0), then we say that the fuzzy graph has no crossing. Then the crossing will not be important for planarity.
- If $\mu(w, x) = 1$ (or near to 1) and $\mu(y, z) = 1$ (near to 1), then the crossing will be important for planarity.
    - Strong fuzzy planar graph if f is greater than or equal 0.5.
    - Otherwise weak.

Definition: 2.18

Let $G = (V, E)$ be a graph. A set $D \subseteq V$ is a $\boldsymbol{dominating\ set}$ [3] of G if every vertex in $V \backslash D$ is adjacent to some vertex in D.

Definition: 2.19

The $\boldsymbol{dominating\ set\ \gamma(G)}$ [3] of G is the minimum cardinality of a dominating set.

Definition: 2.20

A dominating set D is a $\boldsymbol{minimal\ dominating\ set}$ [3] if no proper subset $D' \subset D$ is a dominating set of G.

Definition: 2.21

Let $G = (V, \sigma, \mu)$ be a fuzzy graph on V. Let $x, y \in V$, if x dominates y in G if $\mu(x, y) = \sigma(x)^\wedge\sigma(y)$. A subset D of V is called a $\boldsymbol{dominating\ set}$ [4] in G if for every $v \notin D$, there exists $u \in D$ such that *u* dominates *v*.

Definition: 2.22

The minimum fuzzy cardinality of a dominating set in G is called the ***dominating number*** [4] of G and is denoted by $\gamma(G)$ or γ..

Definition: 2.23

Let G be a fuzzy graph without isolated vertices. A subset D of V is said to be a ***total dominating set***[4] if every vertex in V is dominated by a vertex in D.

The ***total domination number*** of G is denoted by γ_t.

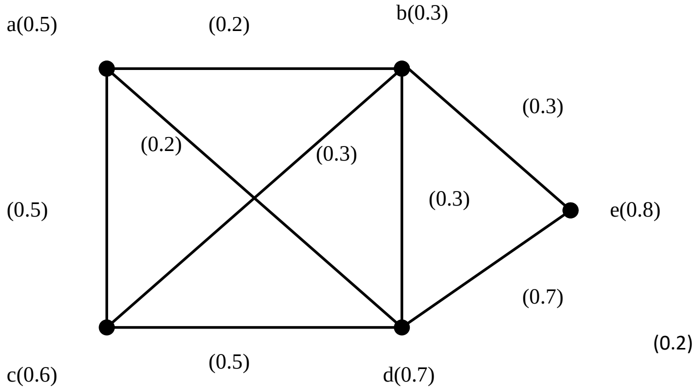
III. DOMINATION IN FUZZY PLANAR GRAPH

Definition: 3.1

If a graph G is said to be ***domination in fuzzy planar graph*** if

- $G = (V, \sigma, \mu)$ be a fuzzy planar graph with planarity value f
- Let $x, y \in V$, x dominates y in G then $\mu(x, y) \leq \sigma(x)\wedge\sigma(y)$
- A subset D of V is called a dominating set in G if for every $y \notin D$, there exist $x \in D$ such that x dominates y.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number of G and is denoted by γ_{FP}.

Example 3.2



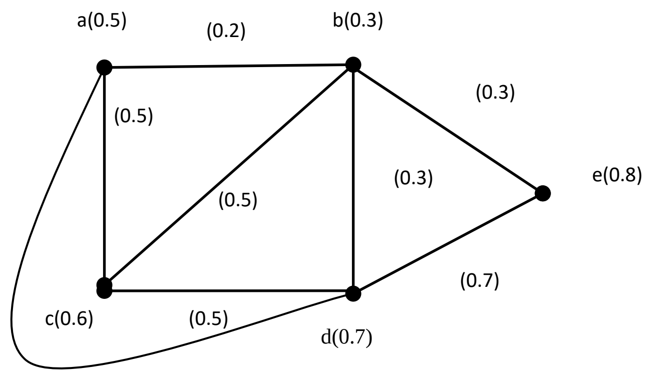


Fig 1. Fuzzy planar graph G_1 with intersection

Fig 2. Fuzzy planar graph G_2 without intersection

G_1 and $G_2$are the same fuzzy planar graph. Planarity value = 0.588

Dominating set = {c,e}, {a,e},{b}, {d}

Minimum dominating number $\gamma_{FP} = 1$

Definition: 3.3

If a graph G is said to *be domination **in strong fuzzy planar graph*** if

- $G = (V, \sigma, \mu)$ be a fuzzy planar graph with planarity value f greater than 0.5
- A subset D of V is called a dominating set in G if for every $y \notin D$, there exist $x \in D$ such that x dominates y.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number of G and is denoted by γ_{SFP}.

Definition: 3.4

If a graph $G_1$and G_2 is said to be *domination in **isomorphic fuzzy planar graph*** if

- $G_1$and G_2 be an isomorphic fuzzy planar graph
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number of G and is denoted by γ_{IFP}.

Theorem: 3.5

For fuzzy planar graph, $p - q \leq \gamma_{FP} \leq p - \delta_E$. Where γ_{FP} be the minimum dominating number in fuzzy planar graph and *p, q* and δ_E are the order, size and minimum effective incident degree of G respectively.

Proof:

Let G be a fuzzy planar . D be a dominating set and γ_{FP} be a minimum dominating number in G.

Then the scalar cardinality of V-D is γ_{FP} and the scalar cardinality of $V \times V$ is $p - q$.

$$p - q \leq \gamma_{FP} \quad \text{……………………I}$$

Let x be the vertex with minimum effective incident degree δ_E. Clearly $V - \{x\}$ is a dominating set and

$$\gamma_{FP} \leq p - \delta_E \quad \text{……………………II}$$

From I and II

$$p - q \leq \gamma_{FP} \leq p - \delta_E.$$

Remark: 3.6

If all the vertices having the same membership value, then $p - q \leq \gamma_{FP} \leq p - \Delta_E$

Theorem: 3.7

Every dominating fuzzy planar graph (with the condition $\mu(x, y) = \sigma(x)^\wedge\sigma(y)$) is the strong fuzzy planar graph according to calculating its strength.

Proof:

Let G be a fuzzy planar graph. Then *x* dominates *y* if $\mu(x, y) = \sigma(x)^\wedge\sigma(y)$.

Hence the membership value of edge will be equal to the membership value of minimum vertex. Then the strength of an edge will be 1.

Strength $I_{(x,y)} = \frac{\mu(x,y)}{\sigma(x)^\wedge\sigma(y)}$

A fuzzy planar graph is strong fuzzy planar graph if the planarity is greater than 0.5.

Then the strength of a strong fuzzy planar graph is 1, then the planarity will be 0.5.

Hence every dominating fuzzy planar graph is the strong fuzzy planar graph.

Theorem: 3.8

Let G be a strong fuzzy planar graph with the minimum dominating set γ_{SFP}.

Then $\gamma_{SFP} \geq p - \Delta_N$.

Proof:

Let G be a strong fuzzy planar graph with the minimum dominating set γ_{SFP}. Let *v* be a vertex such that $dN(v) = \Delta_N$.

Then $V \backslash N(v)$ is a dominating set of G.

Hence $\gamma_{SFP} \geq p - \Delta_N$.

Theorem: 3.9

For any fuzzy planar graph G, total dominating set of a fuzzy planar graph $\gamma_{tFP} = p$ if and only if every vertex of G has a unique neighbor.

Proof:

Let G be a fuzzy planar graph with unique neighbor.

If every vertex of G has unique neighbor then D is the only total dominating set of G.

So that $\gamma_{tFP} = p$.

Conversely, suppose $\gamma_{tFP} = p$.

If there exists a vertex *v* with two neighbors *x* and *y* then, $V - \{x\}$ is a total dominating set of G.

So that $\gamma_{tFP} < p$ which is contradiction.

Hence every vertex of fuzzy planar graph has unique neighbor.

Theorem: 3.10

Let G_1 and G_2 be the two isomorphic fuzzy planar graph with the minimum dominating set γ_{IPF_1} and γ_{IPF_2}. Let f_1 and f_2 be the fuzzy planarity values. Then

a) $\gamma_{IFP_1} = \gamma_{IFP_2}$
b) $f_1 = f_2$

Proof:

Let G_1 and G_2 be the isomorphic fuzzy planar graph.

i.e G_1 is isomorphic to G_2.

Now isomorphism preserves size, weight of the edges and vertex of a fuzzy planar graph.

Hence, weight of the edges and vertex of G_1 and G_2 will be same.

Then we can draw G_1 and G_2 are similarly.

Hence, $\gamma_{IFP_1} = \gamma_{IFP_2}$ and $f_1 = f_2$.

VII. CONCLUSION

This study described the domination in fuzzy planar graph (γ_{FP}), domination in strong fuzzy planar graph (γ_{SFP}), domination in isomorphic fuzzy planar graph (γ_{IFP}). Using the concept of finding the strength of an edge, we defined the strong fuzzy planar. We defined the relationship between maximum neighborhood degree and γ_{SFP}. Then we introduced the relation between the fuzzy planar graph with the planarity and domination in isomorphic fuzzy planar graph.

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